# Investigations on Squash Ball Bounce 

Martin Hirschkorn, PhD<br>24 August 2023

## Background

Squash bounce is a known variable of the game. Balls must be warmed up before a match in order to bounce properly. Even when warm, the ball's bounce depends on many factors such as speed rating of the ball, brand of the ball, age of the ball, room temperature, court conditions, and game style.

This influences preparation for the game (warming up the ball), but also causes a variety in the game based on the ball used and affects the strategy during a game. Eliminating this variety might not be possible (or perhaps desired as it could affect some strategies), but understanding it is important.

The performance of squash balls is defined and specified by the World Squash Federation [1], and the effect of temperature on their behaviour has been previously investigated [2,3,4]. But there are some questions that could be further investigated, particularly with different types and brands of balls:

- How much difference does warming the ball make?
- Is there a difference between different ratings and brands of balls?
- Do balls wear out? What does that even mean?


## Measurements and Calculations

Full details of the measurement techniques and calculations performed are the Appendix, and are summarized here.

The amount that a ball bounces can be expressed as a Coefficient of Restitution (COR), which is the ratio of velocities of the ball before and after a bounce. The COR can be calculated from velocities before and after a bounce (either on the ground or against a wall), or from the time a ball is in the air (hang time) between successive bounces on the ground.

Two methods were developed to measure the velocities or hang times: direct measurement of velocities from high-speed video, and measuring timing between bounces on the floor of a court using a purposebuilt mobile phone app. The second method was used for all results presented here.

As the COR of a ball is not constant at all speeds, a procedure was developed to characterize the behaviour at a single temperature to a single value, representing the COR at $10 \mathrm{~m} / \mathrm{s}$ incoming velocity, denoted by $\mathrm{COR}_{10}$.

There are several factors that can cause significant variance in the measurements:

- Room temperature
- Court wall and floor composition
- Variety in performance of balls, even of the same rating

While the results presented below are intended to highlight differences between various balls, there will be some variance in measurements taken on a different day, in a different court, with a different ball.

## Results

## Ball Temperature

A warm ball is clearly faster (bouncier) than a cold ball, and this first set of tests compares how much.
For these tests, a Dunlop double-yellow ball was tested at five temperatures from 25 (slightly over room temperature) to 45 degrees (a reasonable temperature for a high-level match).

The results can be seen below:


As expected, a warmer ball is significantly faster (higher $\mathrm{COR}_{10}$ ) than a cold ball, and the relationship looks approximately linear.

The COR10 increases from 0.33 to 0.47 across the temperature range, which is an increase of $43 \%$. This means a large difference in gameplay can be expected if a ball is not properly warmed at the start of a game, and between games with players of different levels and styles where the ball temperature does not reach as high a temperature.

## Ball Brand

Tests were performed to compare the behaviour of six different new balls: double-yellow dot balls from three companies (Black Knight, Head, and Dunlop) and three different speed-rated balls (Dunlop singleyellow and red, and Black Knight blue). While the double-yellow balls are intended to perform similarly, the other balls are designed to bounce more.

The results can be seen below.


As expected, the blue and red balls are faster than the others, and the single-yellow ball is faster than the double-yellow from the same company (Dunlop). However, there is a large variance between the three double-yellow balls. Head is significantly faster than Dunlop, and Black Knight is even faster than Head.

It is important to note that these tests show the performance at known temperatures, but no testing was done to see whether the different brands of balls would warm differently, and might reach different temperatures during a match.

## Ball Age

There are many thoughts about how the behaviour of a ball changes as it gets used. Players in professional matches can request new balls during a match, and might make this request because they feel the bounce has changed.

It is also a commonly accepted that balls should be replaced after several matches. If there is a hole in the ball and it deflates, it clearly needs to be replaced. There is also a concern that as the surface of the ball is worn, it becomes smooth and shiny, and will skid or slide more easily, which also changes the behaviour.

But it also seems that after several matches, balls are 'dead', and just don't bounce very much even after playing long enough to warm up the ball. This is tested below. Bounce tests were performed on two Dunlop balls: one new; the other used for several matches, which felt noticeably slower during play than the new ball.


The results show that over the normal temperature range, there was no significant difference in bounce, so this does not explain why the used ball feels slow.

Another possible hypothesis is that while the bounce is the same for an equivalent temperature, the used ball is worn smooth, and the lower skidding friction means the ball does not warm as much during normal play.

## Discussion

This work is not the first attempt to measure and compare squash ball bounce, but it does provide a normalized measure of bounce $\left(\mathrm{COR}_{10}\right)$ that can be used to directly compare the performance of a ball at the given temperature, and proposes a procedure to quickly measure the performance with minimal equipment and setup. The results also provide a reference point for discussion about questions related to bounce.

The debate about what is the right ball to use for a given match is probably as old as the game of squash itself. The advice for players of various levels to use faster balls is rarely followed by players, as players do not want to feel like they are playing a different game than the pros. But as the comparison between different squash ball brands and ratings shows, if the ball temperature during the game is lower than a pro match, the ball is significantly slower; a faster ball will actually behave much closer to the ball in a pro match.

The difference between different brands and balls is significant, but temperature has a larger effect. Switching from a double to single yellow ball is equivalent to just a $5^{\circ} \mathrm{C}$ change in temperature. In other words, a high-level match where the double-yellow ball reaches $45^{\circ} \mathrm{C}$ is equivalent bounce to a match where the single-yellow ball reaches $40^{\circ} \mathrm{C}$.

It is also interesting to note that there is a large variety in behaviour between different double yellow balls, and two brands were found to be as fast or faster than a single yellow ball.

There are several things to consider related to this and possible future work:

- All results shown in a single plot were performed on a single day, but results could vary between days, probably due to court temperature. Care should be taken when comparing results from different days.
- All tests were performed with a single ball. There could be significant difference between different samples of the same brand of ball, due to manufacturing differences or amount of time on the shelf. More samples should be tested before drawing firm conclusions about different brands of balls.
- While this work was all performed with a known, measured ball temperature, the actual temperature the ball will reach during a game will depend on many factors, and changing to a different brand might result in a different temperature.
- It would be interesting to measure ball temperature in different matches at different skill levels, which could inform a decision about the appropriate ball to use.
- These tests measured bounce only, but surface friction (skidding) seems to have a large effect on gameplay also, and could be investigated further.


## Appendix: Measuring Bounce

## Math Background

The bounce of a ball is usually described by the coefficient of restitution (COR), which is defined as the ratio of velocity before $\left(v_{1}\right)$ and after a bounce $\left(v_{2}\right)$ :

$$
\operatorname{COR}=\frac{v_{2}}{v_{1}}
$$

If air resistance is ignored, this can also be calculated from the heights of the ball bouncing:

$$
\operatorname{COR}=\sqrt{\frac{h_{2}}{h_{1}}}
$$

If the ball bounces multiple times, the COR can also be calculated from by measure the time the ball is in the air between two bounces (sometimes referred to as hang time):

$$
\operatorname{COR}=\sqrt{\frac{t_{2}}{t_{1}}}
$$

Another common measurement of bounce is rebound, which is simply the ratio of heights or times before and after a bounce:

$$
\text { Rebound }=\frac{h_{2}}{h_{1}}=\frac{t_{2}}{t_{1}}
$$

This can easily be converted to or from a COR:

$$
\text { Rebound }=\text { COR }^{2}
$$

## Temperature and Velocity Dependence

While the COR definition is very simple, it is not easy to apply it to squash balls, as the COR changes drastically based on two other conditions: temperature, and velocity.

As all squash players know, a ball must be warmed up before playing a game, as the bounce significantly increases with temperature.

For some balls that are made of solid material (solid rubber balls or ball bearings) undergoing small deformations, the COR could be approximately constant at all velocities. That is, the ratio of incoming to outgoing velocity is constant, whether the contact velocity is high or low.

However, this is not the case for squash balls. As the ball is hit faster, the COR decreases. Or more intuitively, if a ball is hit into a wall twice as fast, the rebound velocity will not be twice as fast.

Given these differences, it is not possible to simply define a single COR for a ball. It must be defined for a single temperature and velocity.

This explains why the World Squash Federation (WSF) specifications for rebound of squash balls is defined at 45 degrees (the approximate expected temperature for ball in a professional match) performed at a height of 254 cm .

## Measurement Methods

The previous equations lead to three relatively simple methods for measuring the COR or Rebound:

1. Directly measure the velocity of the ball before $(v 1)$ and after $(v 2)$ contact using a camera or other instrumentation,
2. Drop the ball from a known height ( $h 1$ ) and measure how high it bounces ( $h 2$ ), or
3. Drop or throw the ball at the ground and measure the hang times after the first $(t 1)$ and second ( $t 2$ ) bounces.

Method 1 is the most direct but requires a (preferably high-speed) video camera and calibration of the distances in the video. It can be performed vertically (bounce against the floor) or horizontally (bounce against a wall).

Methods 2 and 3 are simple to perform but are significantly limited in the velocities that can be measured, since high velocities would require very large drop heights. Method 2 requires a known, high point to drop from and a way to accurately measure the bounce height. Method 3 requires only a stopwatch by measuring the time intervals between bounces, though it can be performed more accurately recording a video or audio of the test and reading times of the impacts from the recording.

In this work, two methods were used.

## Video Test

In the video test method, a camera is placed near the front of the court facing a side wall, and the ball is hit against the front wall. A target of known length is placed on the wall in order to calibrate the size of the visible area on the side wall.

The process of measuring the position of the ball each frame and calculating the velocity is done using specialized software (Kinovea).


This produces the velocities $v_{1}$ and $v_{2}$ that are used to calculate COR.

## Bounce Test

The bounce test method measures the hang time between two successive bounces, which can be used to calculate COR. This is done using a specially designed app that detects the sounds of the impacts, determines the times, and calculates the COR directly:


The video test can be used for a wide range of velocities but requires setup and post-processing. The bounce test is quick to run, but the highest velocities that can be achieved are limited by the height of the
court. Also, since it used sound to detect the bounces, it does not work well in noisy environments, which are obviously common in squash courts.

In order to compare these methods, a series of tests was performed. A single ball (Dunlop double yellow) was warmed to 40 degrees, then several bounce tests at different heights were performed. While verifying that the temperature was approximately held at 40 degrees, the ball was then hit against the front wall several times at various speeds. For each of the bounces, the COR was calculated.

In all cases, the ball was warmed by rolling the ball under a foot, and the temperature was measured with a handheld infrared thermometer before each bounce test.

As the following figure comparing incoming velocity to COR shows, at equivalent speeds both methods produce equivalent results, but the video test can be used at higher speeds.


## Characterizing Bounce

The previous figure shows a large difference in COR based on speed, ranging from 0.63 to 0.28 , with likely even more difference if higher and lower velocities were tested. This means it is not possible to use a single COR to describe the behaviour of a ball at a single temperature.

However, after performing the same tests on several balls at several temperatures, the relationship between incoming velocity and COR was always very similar. This means a curve could be fit to each of the plots, to produce parameters that could be directly compared.

After trying various types of curve functions, a power curve was found to work well:

$$
y=A x^{b}
$$

Or in this case:

$$
C O R=A v_{1}{ }^{b}
$$

It was found that $b$ was nearly identical for all tests at -0.36 , and so the bounce behaviour can be wellrepresented by a single parameter, $A$.

Applying this to the first test, it is possible to see how closely the fit curve matches the test results.


The best fit was found with $A=1.097$, which had a fit quality of $R^{2}=0.988$.
Since other ball tests provided similar results, each of those tests can be summarized using a single value of $A$, and it should be reasonably accurate to calculate $A$ from a single-bounce measurements at almost any incoming velocity (though it would be prudent to average several tests to minimize experimental errors).

$$
A=\frac{\operatorname{COR}}{v_{1}^{-0.36}}
$$

One final step was taken. Rather than reporting a value of $A$, which does not relate to anything intuitive, another parameter was chosen to represent the bounce behaviour of a ball, $C O R_{10}$, which is the COR of the ball when the incoming speed is $10 \mathrm{~m} / \mathrm{s}$, as calculated from the fit curve. This can be calculated from:

$$
\operatorname{COR}_{10}=\frac{\operatorname{COR} v_{1}^{0.36}}{2.291}
$$

To summarize, while the actual COR of a ball is heavily dependent on incoming velocity, it is possible to calculate an equivalent COR at $10 \mathrm{~m} / \mathrm{s}, C O R_{10}$, which captures the behaviour of the ball. This parameter can be calculated from a few bounce tests at any reasonable velocity, from approximately $6 \mathrm{~m} / \mathrm{s}$ to $50 \mathrm{~m} / \mathrm{s}$.

The parameter can be used to compare the bounciness of different balls, even if they were not tested at the same speeds. If a ball has a $\operatorname{COR}_{10}$ of 0.48 , it can be confidently stated that it is bouncier at any given speed than a ball with a $C O R_{10}$ of 0.42 .

## References

[1] Specifications for Squash Balls, World Squash Federation, https://www.worldsquash.org/rackets-balls/racket-ball-specifications/
[2] Bounce, Balls, and Surface Temperature, Steve Edgar, http://www.ithacasquash.com/squash/thebounce/thebounce.html
[3] Squash Balls, Squash Player, https://www.squashplayer.co.uk/squash_balls.htm
[4] The dynamic behavior of squash balls, American Journal of Physics, March 2011, G. J. Lewis et al.

